The kinetic energy formula

KE = ½ MV^2

Kinetic energy, KE, is a type of energy that an object possesses as a consequence of its motion. A fundamental rule of energy is that energy is neither created nor destroyed, it's just converted. So, KE = ½ MV^2 really measures the *work* that went into getting that object up to the particular velocity that it's travelling at – where *work* is the application of force over time and distance.

Kinetic energy, one half mass times velocity squared is different to momentum, which is just mass times velocity. So, for example, a car travelling at 60 kilometres an hour needs four times as much braking distance to stop as a car travelling at 30 kilometres an hour – and the 60 kilometre-per-hour car will collide with a brick wall with four times as much force as a 30 kilometre-per-hour car will. However, the car travelling at 60 kilometres an hour only has two times the momentum of the car travelling at 30 kilometres an hour.

To explain this, consider that a lot of work goes into increasing the speed of a car – and work contributes energy. Momentum on its own doesn't tell you anything about the amount of work required to *change* the motion and the momentum of that object. If you want to think about change and what's required to bring about that change – then you need to think about KE = ½ MV^2.

Although it's not really how you derive the math, you can see these principles in the math. So if you apply work to shift an object with a momentum mv up to a faster velocity, its velocity doesn't change instantaneously, it increases steadily over time and over distance. But you can get a measure of the *average* velocity that your work has added, by subtracting the start velocity from the end velocity and then dividing by two. That's where the half in a half MV^2 comes from – kind of, sort of.

To put all this in a thought experiment, imagine a spacecraft flying through interplanetary space, meaning that it's flying through a vacuum *and* it's in microgravity. If you want to increase the spacecraft's velocity – and as a consequence increase its kinetic energy – you will need to activate some kind of propulsion system, which burns fuel. The propulsion system converts chemical energy into kinetic energy – and after the rocket burn is done you could measure what that kinetic energy is using KE = ½ MV^2.

In reality though, a spacecraft's crew has no real interest in what their spacecraft's kinetic energy is – they want to achieve a very specific change in velocity– a delta V. To do that, they would use more complicated math like Tsiolkovsky's rocket equation, to let them to calculate how much fuel they need to burn, and for how long, to achieve a particular delta V – where that math would also account for the ship's mass decreasing as propellant is blasted out of its rockets.

The calculation of kinetic energy has more practical uses when you want to slow something down – particularly if you want to bring something to rest. To do that you will need to apply an equivalent amount of energy, in the opposite direction. Or, if something like an asteroid of known mass and known velocity is barrelling towards you, KE = ½ MV^2 will help you quantify how much damage the asteroid is likely to cause on impact – when all of the asteroid's kinetic energy of motion is converted into some other kind of energy when it suddenly stops being in motion. The actual damage caused by the impact will depend upon where it hits, ocean or land; the angle that it hits at, head-on or a glancing; and what the density of the asteroid is, iron or clay – but knowing what its half MV^2 kinetic energy will be, just prior to impact, will certainly be a useful piece of information.

The Einstein velocity addition formula

$$s = \frac{v+u}{1+(vu/c^2)}.$$

The Einstein velocity addition formula packs in a lot of terms. On one side of the equation S, equals (on the other side of the equation) V plus U, all divided by 1 plus, in brackets , V times U over C squared.

There is a classical interpretation of velocity addition where you imagine yourself on a sailing ship travelling at 20 kilometres an hour – which is around 11 knots. Aboard your sailing ship, you've got a cannon that can fire a cannon ball at 60 kilometres an hour. So, on your ship that's travelling at 20 kilometres an hour, imagine you now point the cannon forward and fire it. For you, the cannon ball will just shoot forward at 60 kilometres an hour – but for someone watching from shore the cannon ball will shoot forward at 80 kilometres an hour, because for them the velocity of the ship (20 kilometres an hour) adds to the velocity of the cannon ball (60 kilometres an hour).

This is how velocity addition works in the context of classical relativity. At least as far back as Galileo, people understood that the measured speed of a moving object will vary depending on the frame of reference from which that measurement is taken. So if you're on the ship, the cannon ball moves forward at 60 kilometres an hour, but if you're on land the cannon ball moves forward at 80 kilometres an hour. Both measurements are completely correct relative to their frame of reference.

So, for classical relativity s just equals V + U – where V is the velocity of the cannonball with respect to the ship and U is the velocity of the ship relative to the shore – and S, the sum of those two velocities, equals the velocity of the cannonball relative to the shore.

So, why the heck does Einstein's law of velocity addition add in so many additional terms, where S equals V plus U, all divided by 1+ V times U over C squared – where C is of course the velocity of light in a vacuum.

In fact, C is the reason for these modifications. In a vacuum, light moves at the fastest speed that anything can move, or be observed to move in our Universe. This velocity C, is about 300,000 kilometres a second. So if you have a ship that moves at nearly the speed of light and it has a cannon that fires cannon balls at nearly the speed of light – no-one in any frame of reference is ever going to see those cannon balls moving forward at nearly twice the speed of light. That's just not possible.

Nonetheless, Einstein's formula does still work in all real situations. So, going back to the ship travelling at 20 kilometres an hour with a cannon that fires cannon balls at 60 kilometres an hour – the cannonball's speed from the shore S, equals V (the velocity of the ball) plus U (the velocity of the ship) which is 80 kilometres an hour. But it’s 80 kilometres an hour all divided by 1 + 60 times 20, which is 1,201 all divided by C squared where C is a bit over 1 trillion kilometres an hour – and a trillion squared is a 1 with more than eighteen zeros after it – so 1,201 divided by such an enormous number equals an exceedingly small number. So 1 plus an exceedingly small number pretty much equals 1. So S pretty much equals V + U divided by 1 – which means that S pretty much equals V + U. This is how it will always work out when we use the velocities we are familiar with in our normal daily lives.

But if the ship was moving 90% of the speed of light and the cannon could fire a cannon ball at 90% of the speed of light, then everything changes. If we say the speed of light C =1, then the velocity of the ship and the cannonball are both 0.9. Putting these in the formula means S = 0.9 plus 0.9 – all over 1 plus 0.9 times 0.9 – which is 0.81. We then divide 0.81 by c squared – but since C is 1, C squared is also 1. So S just equals 0.9 plus 0.9, all over 1 plus 0.81 – which means S equals 1.8 over 1.81 – which works out to be about 0.99. In other words, S equals 99 per cent of the speed of light.

In fact, we can pull out all the stops by imagining that both the ship and the cannon ball can move at the speed of light. This means that S = equals C plus C (which is 2C) all over 1 plus C times C all over C squared. Of course C times C is C squared, so C times C, all over C squared equals 1. So S equals C plus C (which is 2C) over 1 + what works out to be also 1. In other words, S equals 2C over 2 – in other words S equals C.

So, it doesn’t matter what velocity figures you plug into this equation, the answer will never exceed C – the speed of light in a vacuum. And if you still don’t think algebra can be fun, maybe you should run through this podcast again.