De Broglie's Wave-Particle Duality Formula

$\lambda = h/p$

There's no one single formula that captures wave particle duality – it's a principle that extends across a lot of physics, but De Broglie's lambda equals Planck's constant over p is fundamental to the concept. It states that lambda, that signifies wavelength, equals h, Planck's constant, over p which signifies momentum.

That on its own might not signify all that much, but consider that p, momentum, also equals mass times velocity – so that the formula captures a relationship between *mass*, which is something we normally associated with material particles, and *wavelength*, which is something we normally associate with… well, waves.

There's a fundamental problem with the traditional view of atoms where we think of them as a nucleus surrounded by orbiting electrons. Toward the end of the nineteenth century, James Clarke-Maxwell convincingly demonstrated that a charged particle in motion generates electromagnetic waves – that is, light. But, if an electron produced electromagnetic radiation as it moved around and around a nucleus – then, its orbit should steadily decay as a result of the energy it loses through producing that light.

Needless to say, an orbiting electron in a stable atom at room temperature doesn't do any of these things – it doesn't produce light and nor does it spiral into the nucleus as a consequence. De Broglie, indeed Prince Louis-Victor de Broglie, proposed that electrons around atoms are best thought of as waves, even though in other circumstances they might seem to act like particles. So, rather than thinking of an electron as a point-like particle in an orbit, he proposed the whole orbit was in fact a wave.

This suggestion provides a mechanism to explain Max Planck's proposal that thermal radiation from a heated material is released in discrete quanta – like little packets of energy, rather than a continuous flow. We also need to mention Neils Bohr's concept of electron shells here. So, if you heat a chunk of iron, it's starts to glow, because its electrons are absorbing energy and jumping up to a higher energy shell – and then radiating off that heat by releasing a photon, an energy packet, which allows the electron to return to its lower energy shell.

The quantum, non-continuous, nature of the process means there are no intermediate stages in the shift from one energy shell to another. This is the where the term *quantum leap* comes from – an electron cannot exist in-between energy shells. It can jump from one orbital shell to the next orbital shell and drop back again, but it's never to be found in transit between shells. It's there and then it's there.

De Broglie's proposal that an electron is really a wave – explains all this nicely. As a wave, an electron can only occupy an orbit that allows its waveform to remain in phase with itself – in phase meaning has to be peak-trough, peak-trough all the way around, with every peak meeting a trough in an orderly fashion. If you tried to shift the circumference of the orbit, then peak might no longer meet trough all the way around so the wave would be all jumbled up and out of phase. In fact there will only be a few discrete orbits where the whole waveform can stay in phase around a particular circumference.

That's the physical explanation of De Broglie's formula. The math of De Broglie's formula of lambda (wavelength) equals h (Planck's constant) over p (momentum – which is mass times velocity) is derived from some key mathematical and theoretical breakthroughs made over the course of the early twentieth century.

Max Planck demonstrated a quantised relationship between the *frequency* of radiating light and the *energy* of that light – represented as E equals h (Planck's constant) times light frequency. Einstein then showed that light quanta (that is, *photons*) had a certain *momentum*, from his Nobel Prize winning work on the photoelectric effect. And Einstein also demonstrated a certain *mass-energy equivalence* from his work on relativity, where E equals mc squared.

It's beyond this podcast to talk through all the derivation steps in audio– but if you start with mc squared equals h times light frequency, since both sides of that equation each equal E for energy, then you really do end up deriving that lambda equals h on p – remembering that p (momentum) is shorthand for mass times velocity and that lambda, wavelength, has an inverse relationship with frequency. It's all out there on the Internet if you want to try it. Go on…

Newton's Law of Universal Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

Todays' formula is about gravity – and the formula reads as follows: F equals G times big-M times little-m all divided by the radius squared – where that radius is the distance of separation between the centres of the two masses, little-m and big-M. F stands for the force of gravitational attraction and G is our universe's gravitational constant.

This is Newton's Law of Universal Gravitation – and it's pretty useful for many purposes, although no-one considers it to be universal any more, preferring Einstein's theory of general relativity, to provide a more exact representation of gravity. Mind you, Einstein's theory may have its limits as well – for example, at the event horizon of a black hole.

Nonetheless, general relativity more effectively explains that the vector of motion of one massive body is naturally bent towards another massive body, because all massive objects naturally bend spacetime. So for an Einsteinian purist, gravity is not a force, indeed it's not really anything – what we call gravity is just the effect of spacetime curvature influencing the trajectory of things.

But, until Einstein came along, Newton's universal gravitation equation proved very versatile and it remains an excellent way to *approximate* the effects of gravity today. It only begins to fail in close proximity very massive objects – for example, it can't quite manage to model the orbit of Mercury around the Sun – and as for black holes, well, just forget it.

In its day though, Newton's law of universal gravitation was quite something – providing a rigorous mathematical framework to not only model how apples fell from trees, but also how planets orbited stars. And despite the enormous breakthrough that this represented back in 1692, Newton, to his credit, had an inkling that something wasn't quite right when he said: *That one body may act upon another at a distance through a vacuum without the mediation of anything else… is to me so great an absurdity..."*

So, Newton was clearly troubled by the idea that gravity was an invisible force that somehow acted at a distance through a vacuum, but he had no other explanation to offer, since he had no idea that spacetime went all bendy in the presence of mass. If it were somehow possible to introduce Newton to Einstein, Newton might have picked up the idea of general relativity after a quick chat – and if Galileo had then entered the room he probably would have joined in with all the forehead slapping too – provided someone was there to translate everything into Italian. To his credit, Galileo, back in the 1500s, had shown that completely-different masses will all fall at the same rate of acceleration if they are dropped from the same height on Earth.

From Newton's formula, the apparent gravitational force between two masses is inversely proportional to the distance that separates them squared – so that halving the distance between two objects quadruples the apparent force of attraction between them. There is no limit how far that apparent force extends, so at any point in space there will be some measureable degree of gravitational force. This underlies the concept of a *gravitational field*. In Newtonian physics, you would think of an extended field of variable force, while in Einsteinian physics, there's not really a *field* per se, just the underlying *spacetime continuum* and no forces are involved.

It's also interesting that Newton's formula has the constant G, a universal conversion ratio that allows you to determine gravitational attraction, when measured as the product of two masses divided by the square of the distance between them. That same G is also part of Einstein's field equations for general relativity – where it represents the proportional relationship between spacetime curvature and energy density – remembering that mass and energy have an e=mc squared equivalence.

Gee and indeed whizz.