Tsiolkovsky's rocket equation

$$\Delta v = v_e \ln \frac{m_0}{m_f}$$

The rocket equation tells us how the velocity of a rocket will change as it burns a given mass of fuel. Or to put another way, it tells us how much fuel we'll have to burn to achieve that change in velocity.

The equation says that the change in velocity (delta v) achieved by a rocket is the product of the efficiency of its engines (measured as effective exhaust velocity or Ve) and the natural logarithm of the rocket's initial mass (mzero) divided by its final mass (mfinal). We need to use the natural logarithm because, from point of ignition the rocket is burning fuel, and hence losing mass, moment by moment. The natural logarithm is used when you want to calculate change arising from a very large number of small iterative steps.

But to begin at the beginning… Newton's first law of motion, the law of inertia, says that if you are moving, you'll just keep on moving, unless something gets in your way – and if you're not moving… well, you'll just keep on not moving.

So the first thing to understand is that if you are flying a spacecraft through a vacuum in interplanetary space where there's almost no gravity, you will get from point A to point B without having to do anything. Since you are already travelling at a particular velocity, you'll just keep on travelling at that particular velocity. But if you want to get to point B faster then you've got to change your velocity and if you want to launch off a planet then you'll have to change from a surface-relative velocity of zero up to that planet's particular escape velocity.

Newton's second law of motion helps us to understand how you go about changing your velocity. It says that acceleration is a product of the motive force applied to an object divided by the mass of that object. In a roundabout way, Tsiolovsky's rocket equation is actually derived from Newton's second law and represents a special case of the second law– where a motive force is being applied to an object while its mass is steadily decreasing.

And of course the motive force created by your rocket engine is all about Newton's third law. A standard rocket engine creates a controlled explosion that drives propellant out of the rocket's nozzle at very high speed. And if propellant is pushed out in one direction then the rocket will be pushed in the opposite direction – since every action has an equal and opposite reaction.

Rocket engine effectiveness is mostly about engineering. You want to maximise your exhaust velocity and also ensure its goes out in as straight a line as possible, opposite to the direction of motion that you want your rocket moving in. Any misaligned propellant means your exhaust velocity is less effective than it could be.

So, having established all that background, let's now go back to Tsiolkovsky's rocket equation. If a rocket is going to do more than just sit motionless on a planet's surface or to keep moving at a constant velocity through empty space – then you'll need to generate a delta v, a change in velocity.

It's easy enough to work out on paper what delta v you'll need to launch from the Earth's surface up to low Earth orbit, or up to geosynchronous orbit, or to escape from the planet entirely. Armed with that knowledge, the rocket equation tells you what kind of rocket to build, since a rocket is just an aerodynamic fuel tank with rocket engines at the bottom and a payload at the top. You should know what your rocket engines' Ve is – either because you built them, or because you bought them. And you should also know what you'd like your mfinal to be – which will be the mass of your payload, plus the mass of the empty fuel tanks and expended rockets.

The rocket equation also helps you think about rocket staging. For example, if you want to get to low Earth orbit you'll need a delta V of 9.7 kilometres per second. With a single stage rocket, it works out that 88% of your launch mass has to be fuel. But with a two stage rocket, the first stage might only achieve a delta v of 5 kilometres per second, so only 67% of the first stage's mass has to be fuel. The second stage can then achieve the remaining 4.7 kilometres per second of delta V needed to get to low Earth orbit if just 65% of its mass is fuel. And so it works out that your two stage rocket will still get to low Earth orbit, but only 83% of its launch mass needs to be fuel, instead of 88%.

And all that was worked out by Konstantin Tsiolkovsky while living in a log cabin 200 kilometres southwest of Moscow. Tsiolkovsky died in 1935, seven years before the first rocket was launched into space in 1942. That's pretty fantastic.

Hooke's Law

F = kX

Hooke's Law, first announced by Robert Hooke in 1676 is captured in the formula F = kX, where the force F required to stretch a spring is proportional to the distance X that the spring is stretched across. The proportional relationship between F and X depends on the elasticity of the material – represented by K, which is also known as the spring constant. So, in response to the same stretching force a material with a low spring constant will stretch over a long distance while a material with a high spring constant will not stretch as far.

But Hooke's Law really just says that there is a close relationship between force and stretch and the Law only works within certain limits. Hooke's Law is what's known as a first-order linear approximation.

This has nothing to do any recent Star Wars movies but instead indicates that the law only roughly models reality. A first order linear approximation means the relationship described is not precisely linear, but it's not far off

These days there are a myriad of formulas available to describe the specific behaviours of different materials under different stresses. Nonetheless, the relationship F = kX is still useful to broadly define relationships between forces and effects over the distance for which a spring, or some other stretchable material, can maintain its elastic properties – meaning it can tolerate a certain degree of stretching (or compressing) and then spring back. But the degree of tolerance has limits and Hooke's Law can't help you determine where those limits are. So, you first have to determine the points of failure of different materials by experiment and then appreciate that Hooke's Law will only apply well within the boundaries of those points of failure.

For example, through experiment and measurement, we know that timber and steel have lower spring constants than, say, concrete. This means that, well within the boundaries of their respective points of failure, timber and steel may bend in and out of position under stress, while concrete hardly bends at all. So, concrete is often a favoured building material since it will hold its form under considerable stress.

Nonetheless there are contexts where it might still be better to build with timber or steel. Hooke's Law about the relationship between stretch distance and force only applies well within the boundaries of points of failure. If you reach a point of failure with concrete it cracks and shatters with potentially-catastrophic effects. Reaching point of failure with timber of steel means structural components are bent irreversibly out-of-shape and don't spring back, but there may be less mayhem and death as a consequence of that.

Putting construction to one side, Hooke's Law also tells you force that will be generated when a compressed or stretched spring is no longer constrained and it springs back. This is important in many mechanical systems including clockwork systems, which were kind of a big deal back in Hooke's day. So, you wind up a spring and then let it unwind, where energy is released in a regular and predictable way to drive a clock or a music box, or many other clockwork machines that led us to the industrial revolution. Again, because Hooke's Law is just a first order approximation, things that work like clockwork aren't actually 100% accurate and generally need a lot of fine-tuning adjustments. Nonetheless, back in Hooke's day, this was pretty-impressive accuracy.

When Robert Hooke announced his law, back in 1676, he did it as an Latin anagram – ceiiinosssttuv, which he subsequently unscrambled into Ut Tensio, sic vis – which translates to as the extension, so the force. Back in those days, this is how you announced a fantastic physics formula.