The Maxwell-Faraday equation

$$\nabla \times \vec{\mathbf{E}} = -\frac{d\vec{\mathbf{B}}}{dt}$$

As usual, we'll start by saying what the formula states – and then we'll try and explain what the heck it all means. So, Maxwell-Faraday equation states how an electric current, or more specifically the electromotive force that drives an electrical current, can be induced by an adjacent magnetic field that changes with time. But, literally, and even more inscrutably, what the formula really states is that the curl of an electric field represented by an upside-down triangle multiplied by E is in the opposite direction to the change in magnetic flux over time – which is represented by a minus sign in front of dB over dt.

OK, now we'll try and explain what the heck all that means. Michael Faraday, who was English, was an experimenter more than he was a theorist, though he did his best to pull together a robust theoretical schema around the experiments that he did. James-Clerk Maxwell , who was Scottish and was very much a theoretician, applied Faraday's experimental findings to describe how electricity and magnetism are really two aspects of the same thing – what we today call the electromagnetic force.

So, Faraday's work underlies how we now generate electricity to power our everyday lives. Maxwell's work underlies how we know that an electromagnetic field pervades the Universe and that perturbations in that electromagnetic field are communicated across the Universe as electromagnetic radiation – that is, light, across all its frequencies from radio waves to gamma rays

Although Faraday's name comes second in the formula title, it was his work that kicked off all this thinking. One of Faraday's key experiments was the induction coil. Imagine you move a magnet in and out of a hollow cylinder that has a coil of wire wrapped around it. As the magnet moves in and out of the coiled wire cylinder, an electrical current will be induced in that coiled wire. This is somewhat fantastic itself, since the magnet and the cylinder never touch, so the current is being induced by an effect transmitted through empty space. Well OK, there might be air filling that space, in a standard Earth-based laboratory, but that air plays no role in the transmitting the effect.

What the Maxwell-Faraday formula is trying to tell you in plain English is that an electromotive force (also known as voltage) can drive a current in an adjacent and spatially-separated coil of wire. That electromotive force is induced by a *change* in conditions. You can place a powerful magnet inside a coiled wire cylinder, but nothing will happens unless you move the magnet, or you move the cylinder. Alternatively, rather moving anything you can just change the magnet's flux, which is easily done if it's an electromagnet.

Most electrical generators are built around this concept of change. In a standard generator, you have a *stator*, a stationary wire coil, and a magnetised *rotor* that spins rapidly around the stator to produce an electrical current. The rotor's spin can be driven by falling water, or waves, or wind – or it can be driven by steam that is created by heating water, with burning coal or with nuclear fission.

In fact, the Maxwell Faraday formula is essentially a description of such power generating processes. If you change magnetic flux over time, perhaps by spinning a magnetised rotor, then that change induces an electric current. The current induced moves perpendicular to the plane of rotation of that rotor, but the current itself produces its own magnetic field perpendicular to the motion of the current. So the magnetic field that induced the current and the magnetic field arising from that current are in opposition to each other – which is why the formula has a minus sign on one side of it.

If that's not confusing enough, consider that a moving electrical field creates a perpendicular magnetic field that creates an inverted electrical field that creates an inverted magnetic field, all of which creates a wave that can self-perpetuate its way through a vacuum. This is how electromagnetic radiation works – a wave that propagates through nothing.

Albert Einstein was clearly intrigued by Maxwell's insights into Faraday's experiments. Indeed Einstein spent the whole first paragraph of his first paper in 1905 on special relativity, *The electrodynamics of moving bodies*, describing the paradigm-shifting implications of the Maxwell-Faraday equation. So, if all that doesn't seem even a tiny bit fantastic, it's maybe time that you updated your podcast subscriptions.

Ohm's law

$$I = \frac{V}{R}$$

Ohm's law is refreshingly simple in containing only three terms and although it was derived from early experiments with the flow of electricity, its general principles can be applied to other contexts – like the flow of fluids, which we'll get to later. The formula states that I (current) equals V (voltage) over R (resistance). Or you can just say that the formula shows how current is directly proportional to voltage – and that proportionality is related to the resistance in the electrical circuit.

So, for example, if you apply a certain voltage between two points connected by a wire, there will be a particular flow of electrons along that wire between those two points. If you apply a higher voltage, there will be a higher electron flow, if you apply a lower voltage there will be a lower electron flow.

But, you can change the wire – say from copper to aluminium, which will then change the resistance. In that case, current will still increase as voltage increases, but the ratio between voltage and current will be different. You could also keep the same wire, but change its temperature – since the electrical resistance of most materials goes up when hot and goes down when cold. So, at different temperatures current will still increase as voltage increases, but the ratio of that effect will be different at different temperatures.

So, while we say resistance represents the ratio between voltage and current – it's more like a third player in the game. For example, in most household electrical circuits, voltage is generally constant and you add a variable resistor into a circuit, like a volume control or a light dimmer. Then, by changing the resistance, you change the current flow.

For most household-level electrical design scenarios, you'll already know the voltage being delivered and you know the current that different appliances need to run at, so design work is mostly about ensuring you have the right resistance in the circuit – and that's where the standard formula of I (current) equals v (voltage) over R (resistance) is useful. On the other hand, you might not have the faintest idea what the resistance of your circuit is – but if you know the values of voltage and of current then you just flip the formula around so that R (resistance) will equal V (voltage) over I (current). It's also true that V (voltage) equals I (current) times R (resistance).

And if that's not fantastic enough, Ohm's law works also beyond the physics of electricity. If we keep the concept of current but replace the concept of voltage with some other driving force, then Ohm's law also works in the world of plumbing. So, current – that is the flow of water molecules from point A to point B – is directly proportional to the driving force between point A and point B.

That driving force will be a pressure differential between points A and B, perhaps created by a pump, or by gravity, or even by heat. But, while flow will always increase in face of an increased driving force, but you won't get as much flow through a narrow pipe as you will through a wide pipe, as the wide pipe will offer less resistance. So, it's the resistance to flow that determines the ratio between the driving force and the current flow.

So, there is an extent to which the flow of electricity follows the principles that govern the flow of fluids. Or maybe it's the case that the flow of water follows the principles that govern the flow of electricity. Either way, there's some vastly more complicated formulas that deal with the hydrodynamics of different systems and some vastly more complex formulas that deal with the electrodynamics of different systems – but it is kind of fantastic to discover that there's some consistency in the way that different things work in the Universe – at least at a superficial level.