Heisenberg's uncertainty principle

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

Being a principle, there are a number of different ways in which Heisenberg's uncertainty principle can come into play. The best known example is that the more you try to hone in on a particle's position x, the less you are able to determine its momentum p. And if you instead devote your efforts towards determining its momentum, you will find your ability to determine its position beginning to slip away. Essentially, you can never know either of these aspects of the particle's behavior exactly and the more precisely you know one, the less precisely you can know the other.

In fact there are a range of paired characteristics of particles that you can never know at the same time – their position and momentum, their energy and duration, their different axial spins, their entanglement and coherence and of course their wave and particle characteristics. It's wave-particle duality that underlies Heisenberg's formula, which is why a modified form of Planck's constant h*bar*, is there. If you go back to our episode on de Broglie's wave-particle duality formula, it's Planck's constant that defines the proportional relationship between the wavelength and the momentum of fundamental particles like electrons

Anyhow, in respect of a particle's position and momentum, Heisenberg's uncertainty formula says that the product of the standard deviations of position, sigma x and momentum sigma p must be greater than or equal to Planck's reduced constant over 2. Standard deviation is a statistical representation of the precision of a measurement. So, if you measure the same thing a hundred times over and you get exactly the same answer each time then the standard deviation is zero. But generally whenever you measure natural phenomena there's likely to be a bit of background noise or perhaps your measurement instrument isn't as finely-tuned as you'd like it to be. So, those 100 measurements won't all be the same even though they may hover around one point of central tendency. If all your repeated measurements are close to one central point then your standard deviation is low, which means your central measurement is of high precision, even if it might not be exact. On the other hand, if your repeated measurements are widely spread then your standard deviation is high which means your measurement precision is low.

But of course, this is quantum mechanics. At the quantum level, there is an inherent indeterminacy about the things that you are measuring. So the fact that you can't exactly nail the measurement you are trying to make isn't just about background noise or the precision of your measuring instruments, you intrinsically *cannot* nail the measurement.

Perhaps the best thing to say is that today's science stands on the shoulders of giants. Clever people in history long ago worked out the physics of oceans waves and tennis balls and today we find their fantastic physics continues to work, even in the new world of subatomic physics. It is a credit to our modern-day physicists and mathematicians that this is even possible, but we have to remember that at the subatomic level we are not dealing with oceans waves or tennis balls. For the most part, we don't really know what we are dealing with – but it's a triumph of the human intellect that we've found a way to model whatever is going on down there by overlaying some statistical formulas about measurement precision onto some physical formulas about ocean waves and tennis balls.

In a couple of hundred years everyone may be swapping jokes about those quaint 20th and 21st century folks who thought everything was both waves *and* particles, although they weren't exactly sure by how much. So, there may still yet e some deeper explanation that will make our future selves giggle about how we missed it, but right now this really is what the quantum world looks like.

So, there is always an imprecision around any measurement of position and an imprecision around any measurement of momentum. Those imprecisions are shown as standard deviations and according to Heisenberg's principle the product of those two standard deviations always has to be greater than or equal to hbar on 2.

The right-hand side of the formula is hbar on 2 because the formula is derived from other formulas like de Broglie's wave-particle duality formula. Hbar on 2 is a very small number and represents a threshold below which the left-hand side of the formula just can't go. The left-hand side of the formula is the standard deviation of x, position, times the standard deviation of p momentum. So, to stay above the threshold, neither standard deviation can reach zero, because if one term is zero, then zero times anything is still zero – and a standard deviation of zero means absolute precision – which just can't happen. Indeed if the standard deviation of one term starts approaching zero (precision), then the standard deviation of the other term has to grow (imprecision) to ensure the left hand side of the formula stays above hbar on 2.

So, *is* Heisenberg Uncertainty Principle a fantastic physics formula? Heck, yes. And *is* the Universe really as uncertain as the formula makes out? Well, probably.

The wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

Since this is Fantastic Physics Formulas, we'll first tell you what the formula is and then we'll spend the rest of the episode trying to explain what the heck it means. So, firstly the formula says that the second derivative of u with respect to time is equal to c squared times the Laplacian of the wave at u.

If you remember our earlier episode of Fourier transforms, a mathematically useful way of portraying a wave is to turn it into a function that's drawn out on an x y axis, where x is time and y shows the amplitude of the wave – that is, how the wave goes up and down over time. If we start by imagining that u is a point in space that can move up and down with respect to time, then at different points in time, u might be at the top of the wave or at the bottom or somewhere in between.

If you calculated the first derivative of u that would tell you how u moves over time, so you get a picture of the velocity of u at a particular point in time. But, if you calculate the second derivative of u you are then measuring how velocity changes over time – that is, the acceleration of u.

For example, imagine a buoy on the surface of the ocean that's rising and falling with time as waves pass it by. As a wave hits, the buoy undergoes rapid acceleration upwards and downwards, but over that whole cycle it never actually moves forward or backward.

So, in the wave equation, the left hand side of the equation is about the acceleration of the buoy, where the buoy is the point u. The right hand side of the wave describes the curvature of the wave – which is the Laplacian multiplied by a constant (c^2). When you see a constant on one side of an equation, you know there's an underlying relationship of proportionality. The c in the formula represents the speed of propagation of the wave. c doesn't *have to be* the speed of light, it's just the propagation speed of whatever kind of wave you are considering. But, if you are considering a light wave, then c is the speed of light in a vacuum and the wave equation will work just as well in a vacuum as it will work in modelling oceans wave and sound waves and any other sort of waves.

So, the equation is really telling you that the acceleration of the rising and falling buoy is proportionally related to the curvature of the wave and that proportionality is equal to the square of the wave speed.

In the formula, the Laplacian, denoted by an upside triangle squared is mathematical short hand that can describe the curvature of the wave in multiple dimensions. It's easy enough to imagine a three dimensional wave by imagining you drop underwater depth charge so that when it goes off the detonation wavefront spreads out in a sphere. We use the Laplacian for a 3D wave as it saves us writing the curvature in the x direction + the curvature in the y direction + the curvature in the z direction. The first wave equation developed back in the 1700's, just dealt with one dimension – and so it did just state that the curvature of a wave in one dimension was the second derivative of u with respect to x. So the wave equation in one dimension just describes a wave in the horizontal dimension, represented by the x axis. A wave equation in two dimensions would also include the second derivative of u with respect to y, representing the vertical dimension and then a 3 dimensional spherical wave could be described by also including the second derivative of u with respect to z – which is like the wave is coming out of the page right at you

So, the wave equation is really a multidimensional tool. The one dimensional wave equation developed back in the 1700s just described a wave on a string. But after we nailed the basic physics of a wave in one dimension, the equation for a spherical wave in three dimensions soon followed – and these days we dealing with string theory which models wave vibrations in either eleven or in 26 dimensions, depending on which particular school of post-quantum string theory you align with. It is all just mathematical modelling, but it does sometimes work out that when we capture data about the real world, it is in agreement with a mathematical model that we've built. And when that happens, we know we've really nailed something – and that is kind-of fantastic.