$E=mc^2$

E=mc squared is a nice straight-forward formula, indeed the energy-mass equivalence concept that underlies it is pretty straight-forward too. So, part of its popularity as the world's most famous physics formula is that everyone kind-of gets it - energy equals mass times a constant – well OK a constant squared, but that is still a constant. But why is that constant a velocity – and why is it squared?

So imagine a spherical flashlight that emits radiation equally in all directions. Switch it on to produce a flash of light, then you know it has lost energy in that event. So straight away you have an equation. The energy now equals the energy then minus the energy that was lost in the flash. Great work, that is a formula, but it is just stating the bleeding obvious. To make a real breakthrough you need to find a new perspective on the situation, ideally one that tells you something new and fundamental about the Universe. Einstein did this by applying the principles arising from his work on special relativity.

So imagine you're an observer that flies past the spherical flashlight at speed just as it turns on. Relative to your frame of reference the flashlight is now moving and therefore it has kinetic energy, which is KE=1/2 mv squared. After it emits the flash, which carries away energy E, the flashlight's remaining energy will be KE minus E. But you also notice that since the flash light is moving, it's energy has changed due to the Doppler effect, so while you're approaching the flashlight it's blue-shifted (more energy) and while you're receding from it it's red-shifted (less energy). This is just classical physics at work, but Einstein added another layer on this, called the relativistic Doppler effect, where not only has the frequency of light changed due to relative motion, but time dilation and length contraction effects are also in play.

So a more correct representation of the energy the flashlight lost in the flash is to overlay a Lorentz transformation that accounts for observational changes occurring across two frames of reference in relative motion to each other. Remember your dealing with a relativistic Doppler effect, so the change in energy is about whether you are approaching or receding from the flashlight when you measure the energy of the flash, that is E, which you'll need to adjust with a Lorenz transformation, which in this case is $1 + v^2/2c^2$.

So by reconfiguring the context of the flashlight releasing energy within a situation containing kinetic energy and relative velocities, you get a formula that includes mass as a component of kinetic energy and the constant c squared as an outcome of applying a Lorenz transformation to account for measurements being made between different frames of reference moving at different velocities . If you then write out the fairly long equation of the changed energy observed from the relative motion of the observer and then cancel out all the things that can be cancelled out, which is mostly all the Vs since the velocity of motion never changes – then all you are left with is E=mc squared. This was mostly stated in Einstein's special relativity paper *On the electrodynamics of moving b*odies in September 1905 and more specifically stated in another 1905 paper *Does the inertia of a body depend upon its energy content?* which specifically explained how mass-energy equivalence must exist even without calling upon the relative motion perspective to prove it through algebra.

Energy mass equivalence is often illustrated by a nuclear bomb blast as though that represents what happens when you release the energy that is normally frozen within mass. But really, the energy and potential explosiveness of nuclear fission is more about releasing the binding energies that hold very large atoms together so they fall apart into smaller atoms. In that process you don't really lose any matter – you pretty-much still have the same collection of protons and neutrons that you started with – you just release a huge amount of heat that was formerly the nuclear binding energies that held those particles together and added mass to the very large atoms those particles formed. So, if you explode an atomic weapon within an invulnerable sealed box, the mass of the box doesn't change at all. But if you open a window in that box and allow electromagnetic radiation to escape, then the mass of the box will decrease. This is an important thought experiment to remind you that not only does mass have energy, but energy has mass.

Einstein's Field Equations

$$G + g\Lambda = \frac{8\pi G}{c^4} T$$

The Field Equations are the technical parts of the Einstein's General Theory of Relativity and allow calculations of how the geometry of spacetime is affected by the distribution of the mass and energy within it.

The Einstein field equations can be stated in one formula which summarizes ten underlying equations. The summary formula gives you a conceptual understanding of what the equations are about and the ten underlying equations allow you to calculate the dimensions and characteristics of a specific spacetime field.

The summarized equation is Big G, plus small g times a constant equals Big T times a constant. Big G is Einstein's tensor which is said to describe a *manifold*, essentially a shape than can be curved and warped. The small g is the metric tensor which contains measurement rules to tell you how to measure the gaps between close and distant coordinate points on the manifold. So, taken together left-hand side of the equation represents how spacetime geometry is shaped and measured.

On the right-hand side is Big T, the energy-momentum tensor, representing the distribution of mass and energy. Remember energy is mass and mass is energy, so this tensor can represent the effect that matter can have on spacetime, by virtue of the mass of that matter, although it can also be about the mass inherent in energy.

So, this is the underlying idea of the field equations, space time geometry (left-hand side) is determined by the distribution of mass and energy (right-hand side), although you could just as well say spacetime geometry determines the distribution of mass and energy. As John Wheeler famously said: *Spacetime tells matter how to move; while matter tells spacetime how to curve*.

And remember there are some constants in equation. The full equation is Einstein's tensor Big G plus the metric tensor, small g times a constant called lambda equals the energy-momentum tensor, Big T times a constant called K. K is sometimes called Einstein's constant and is actually a collection of different constants that come from the underlying math of the field equations. K in full is actually 8 times pi times Newton's gravitational constant G all divided by c to the fourth power, where c is the speed of light in a vacuum. So, within K there's a constant related to geometry, pi, another one to gravity, G and another representing the relationship between space and time, c.

Lambda is the other constant in the formula. It is famously known as the cosmological constant. In Einstein's first 1915 publication on general relativity lambda wasn't mentioned. The paper just proposed the relationship between space time geometry, big G and energy mass distribution, big T. Lambda, along with the metric tensor small g, first appeared in a 1917 paper of Einstein's. These additional terms were proposed to offer a way to calculate the nature of the Universe, or at a least *a Universe*, since the nature of the Universe would change depending on what values you assigned to those terms. Ironically, Einstein introduced Lambda and the metric tensor to support the assumption that the Universe was static and unchanging. This was after a number of mathematicians, notably Friedmann had pointed out that the original formula would naturally see Universe changing in volume over time, so Einstein used Lambda to try and stop that. After Einstein learned the Universe really was expanding, he wrote a paper with De Sitter which did present a model for an expanding Universe, but without the need for lambda or the metric tensor.

Lambda has since been adopted as part of our current Standard Model of the Universe, known as the *Lambda Cold Dark Matter* model, with Lambda essentially representing the cause of the accelerating expansion of our Universe – whatever that may be. This is sometimes said to indicate Einstein was right after all, although for the most part we are adopting something he'd decided was a mistake. Of course, the *Lambda Cold Dark Matter* model is just our latest standard model for the Universe – so it may or may not last. But whatever model we run with, Einstein's fantastic field equations are likely to be part of it.