The Hubble – Lemaitre Law

$$v = H_0 D$$

V (Velocity) equals D (distance) times H-nought (the Hubble-Lemaitre constant).

This formula describes the relationship between the distance of galaxies and the velocity at which they are receding from us. Put simply it says the farther away a galaxy is, the faster it is receding. This relationship has been widely accepted as evidence that the Universe must be expanding, with the notable exception of Edwin Hubble who was happy there was a relationship between the redshift of galaxies and their distance, but never really accepted it as evidence that the whole universe was expanding.

Perhaps the most intriguing part of this formula is H-nought, the Hubble-Lemaitre constant. This is not a natural constant like c, it just quantifies the proportional relationship between distance and recession velocity. To work out what it is, we collect lots of data about the distance of different galaxies and their recession velocity. Within that data we find that a galaxy about one megaparsec away is receding from us at 70 kilometres a second, while a galaxy about two megaparsecs away is receding from us at 140 kilometres a second and so on. In other words, H-nought is about 70 kilometers per second per megaparsec – meaning it's not just a number, it's an expansion rate. So right now, everywhere in our Universe, each megaparsec is expanding by 70 kilometres every second.

But, while H-nought is at any one instant the same everywhere in the Universe, it is gradually changing over time, so it's not really a constant at all and is more often called the Hubble-Lemaitre *parameter* these days. It's actually been declining over the life of the Universe, which is generally explained in terms of the initial high density of the Universe's contents drove a fast rate, which then declined as the Universe's contents were diluted within the growing volume of the expanding Universe. Although this *unit rate* of expansion is declining the overall Universe's expansion still accelerates, as long as H nought persists up to a certain level. It's that persistence that surprised everyone when, back in the 1990s, we discovered the Universe's expansion was still accelerating, with most people assuming such acceleration might have happened early in the Universe's history, but would surely have stopped by now. And so, since that discovery, cosmologists have proposed that there must be something else sustaining the expansion rate, that something being given the working title of dark energy.

We've already mentioned that Edwin Hubble had doubts about the Universe expanding. The other famous expansion skeptic was Albert Einstein. Mass and energy densities have a natural tendency to gravitate together and General Relativity tells us they will pull spacetime in along with them. So either a Universe should be collapsing down on itself or it must expanding so as to avoid that collapse. The only alternative is to imagine a static universe governed by some kind of steadying influence that keeps it eternally on a knife's edge between collapse and expansion. Nonetheless, when Einstein considered the cosmological implications of General Relativity, this was the option he went for in a 1917 paper, plonking his cosmological constant into his General Relativity field equations to make the universe static.

But in 1924 first Alexander Friedmann and then Georges Lemaitre argued that Einstein's field equations indicated that our Universe must be expanding. Indeed, before Hubble had published data showing the relationship between distance and redshift, Lemaitre had already proposed that same relationship. However, he published his conclusions in an obscure French journal which went largely unnoticed. It now seems clear that not only did Lemaitre discover what was called initially Hubble's Law before Hubble did, he also arrived at a much better estimate of H-nought, still often referred to Hubble's constant – while Hubble's initial estimate of H-nought is best described as 'way off', although he did later improve on it.

One other implication of the Hubble-Lemaitre Law is that if the Universe is expanding, then surely it must have been smaller in the past. Indeed, it could well be that the whole thing started from one single point. And who first thought of this astounding idea? Georges Lemaitre, who proposed the idea of a *primeval atom*, some tiny thing which disintegrated in an explosion that gave rise to the whole expanding spacetime Universe we know of today.

So, even if he only gets second-author credit for the Hubble-Lemaitre Law, Georges Lemaitre is acknowledged as the originator of the Big Bang Theory – and that is pretty fantastic.

The Lift Equation

$$L = \frac{1}{2} \rho v^2 S C_L$$

In 2021, we saw some billionaire-owned space flight companies commence passenger flights, along with some debate about what altitude above Earth represents the start of space. The altitude at which the Earth's atmosphere stops is difficult to define, since you could still come across the odd atmospheric particle out beyond the Moon's orbit.

A better definition might be the line at which aerodynamic flight stops and spaceflight commences. This is not only important for air and space flight companies but also for governments, who can claim jurisdiction within their airspace, but not in the space above it.

A commonly accepted boundary between air and space is the Karman Line named after Theodore von Karman. von Karman originally proposed an altitude of 270,000 feet, around 82 kilometres , which has since been adjusted by others to 100 kilometres. So, as it turned out, Virgin Galactic passed the original but not the adjusted line, while Blue Origin passed both. Of course in the new age of passenger flights to space it's no longer enough just to cross either line to earn the title astronaut. In July 2021, the US Federal Aviation Administration added an extra requirement – that you had to be part of flight crew before being awarded astronaut wings. This seems reasonable, aircraft passengers don't get a special badge just because they've been up in a plane.

von Karman's thinking was that the edge of space should be an altitude at which it is no longer possible to gain lift from aerodynamic flight. Lift is the force that pushes an aircraft up and for that aircraft to leave the ground that force has to be sufficient to counteract the aircraft's weight. The various factors that contribute to lift are captured in the Lift Equation.

L equals half rho v squared times S times C subscript L, which in plain English roughly translates as

Lift equals half of the air density, times the velocity of the aircraft squared, times its wing area, times its lift coefficient.

The lift coefficient captures design issues such as the aircraft's geometry and how it interacts with airflow. The lift coefficient is highly scalable, which is why testing small models in wind tunnels is a very effective way to design full scale aircraft.

Maintaining lift at high altitudes isn't just an issue of air density, since you can keep on gaining lift in lower air densities by just going faster. von Karman reasoned that there should be an aerothermal limit – that is, a maximum altitude achievable without exceeding a velocity that would overheat the aircraft's surfaces. von Karman's logic was based on 1950s technology – with more modern materials, the overheating issue has become largely irrelevant and the limit is the altitude at which you have to go so fast to achieve lift that you achieve orbit.

Either 80 or 100 kilometres are pretty extreme altitudes where the air density is around one thousandth of what it is at sea level. So most aircraft are unable to achieve lift well before these heights. Most commercial passenger aircraft have maximum altitudes in the 12-15 kilometre range, although the supersonic Concorde did manage 18 kilometres. Military jets can't manage much more than 30 kilometers altitude– since there's not enough oxygen at higher altitudes to burn their fuel . To go higher than that you need something like the X-15 rocket plane, which gained speed and hence lift using an ammonia-based fuel mixed with liquid oxygen. The X-15 achieved 95 kilometers altitude after being lifted to around 14 kilometres by another plane. The current altitude record for a rocket-powered plane is 100 kilometres, held by VG's Spaceship One, which was launched from another plane at about 15 kilometres, fueled by what was essentially tyre rubber (the fuel) and laughing gas (the oxidiser).

But, it's unlikely you could get into orbit with these kinds of rocket planes because you need a lot more speed. Rockets like the Saturn V are essentially giant fuel tanks with big rockets at the bottom and a tiny payload at the top, aiming to achieve both an altitude of over 100 kilometres and an orbital velocity of over 25,000 kilometres an hour, compared with the X 15's maximum speed of just over 7,000 kilometres an hour.

Given the amount of fuel you need to start with, the current preferred method is to just blast your way up and into orbit. So rather than worrying about the Lift Equation, it's all about Tsilokovsky's rocket equation. Fortunately, that is also a pretty fantastic physics formula.