The orbit equation

$$v_o = \sqrt{\frac{GM}{r}}$$

There are several orbit equations around, but this orbit equation tells you what orbital velocity you need to maintain an orbit at a particular altitude. So v for velocity equals the square root of G, the gravitational constant times M, the mass of the planet you're orbiting, divided by R – which is the radius of the planet plus your altitude above its surface. The planet can be any planet, since G is a universal constant, you'll just need to know the planet's mass and radius, which reflect its density and hence how much gravity it can generate.

The orbit equation is based on Newton's second law where F (force)=MA (mass times acceleration) and it also draws on Newton's universal gravitation formula, where the gravitational force between two masses equals G, the gravitational constant times the product of two masses divided by the square of the distance between them.

The orbit equation can be derived from F=MA since we can deduce that for the spaceship to remain in orbit the force induced by the gravity of the planet at that altitude must be balanced by the spacecraft's motion. So you start with a formula where Newton's Law of Universal gravitation is met by a trigonometric representation of your spacecraft's circular trajectory, which is an angular acceleration. Since your spacecraft's mass is a component of the gravitational side of the equation (involving two masses separated by distance) and it's also a component of the orbital side of the equation (its mass and its acceleration), you can just cancel its mass from both sides, which simplifies the equation down to V squared equals the product of G, the gravitation constant times big M, the mass of the planet, all divided by R the radius of your orbit – which as we said before is the radius of the planet plus your altitude above it. Then take the square root of both sides and you get v equals the square root of G times M divided by r. From this you can see there's an inverse relationship between v and r – that is, if you increase your altitude, the required velocity to achieve orbit decreases and vice versa.

Achieving orbit is a curious business, since you have to accelerate to rise up to it and if you overshoot and start approaching escape velocity you'll have to fire your retrorockets to decelerate back down, but once you are in orbit you can just switch your engines off, potentially coasting forever at the constant velocity that is determined by the orbit equation. This is because you are constantly falling in a gravity field but your velocity ensures you always fall around, rather than towards the centre of that gravity field.

However, in reality, you can't switch off your engines forever. You will need the occasional correction burn to counter any residual air resistance and also to counter any gravitational drag, since you are not orbiting a perfect sphere with a uniform density and that imperfect sphere is rotating beneath you. Potentially you can orbit a planet at any altitude, but flying close to the surface makes it very hard to maintain the needed constant velocity for orbit, since you get more a lot more air resistance near the surface and also things like mountains start becoming an issue. So, successful long-term, low-energy orbits are best achieved at high altitudes.

Anyway, once you decide on your preferred altitude for a sustainable orbit, the orbit equation allows you to calculate the necessary velocity required to orbit at that altitude. Then you can use the rocket equation to design a spacecraft with enough fuel so that it can achieve the required velocity for orbit after it launches from the surface.

As we said before, it's the distance from the centre of the Earth that really matters, but since the radius of the Earth is reasonably constant, it ends up being mostly about your altitude above the surface. So, that's the orbit equation, the very height of fantastic physics formulas.

Entropy and the Inequality of Clausius.

Getting to this formula requires a bit of a story, but it's worth it as it ends up explaining which direction the Universe is moving in. The conversion of heat, a form of energy, into work dates back to ancient Roman times, although heat engines built to do proper industrial work first appeared around the start of the 17^{th} century, mostly steam engines that first pumped water out coal mines and then later drove locomotives, as well as driving a range of other piston-driven machines. Along with all that engineering came a lot of theoretical thinking about how to make such heat engines work more efficiently. A Carnot engine is a thought experiment intended to demonstrate the essential thermodynamic principles underlying the operation of heat engines.

So imagine a piston in a cylinder containing a volume of gas. If you heat the cylinder the gas expands and pushes the piston down, if you then you cut the heat the gas cools and the cylinder moves back up again. Then you repeat that cycle and voila, you have yourself a heat engine. An important detail in all that is the step where the gas cools – which is has to do for the piston to come back down. So you added heat to the gas and then it lost heat. That heat has to go somewhere, so an idealized Carnot engine has a heat source on one side of the cylinder and a heat sink on the other – essentially something cooler that the heated gas can give up its heat to.

So firstly, this is an example of how you need thermal disequilibrium to convert heat energy into work. If the gas is as hot as the heat source then the engine won't work, you first need the gas to be cooler so it heats up and expands, then the gas has to cool down again so that the engine can complete a full cycle. So, essentially the engine doesn't work by consuming energy, it's just positioned in the middle of a heat transfer process from the heat source to the heat sink. This potentially means, in thermodynamic jargon, that the engine is fully reversible. So the heat it absorbs and then later gives up could be re-used to drive the same engine cycle over and over.

Rudolf Clausius argued that while the sum of all transformations from heat to work and then work to heat to complete the reversible Carnot heat engine cycle *could* be zero – for any engine not operating at 100 per cent efficiency the sum of all transformations would not be zero.

He captured this in a formula which states that over each cycle of the engine, the integral of all changes in temperature within the engine, delta Q, divided by the final temperature of the heat sink, T, is either less than or equal to zero, a finding that history now remembers as the inequality of Clausius.

$$\oint \frac{\delta Q}{T_{\text{surr}}} \leq 0$$

It's worth saying here, that no-one, including Clausius thought the sum of all transformations in a heat engine could ever *really* be zero. What Clausius was saying was that the sum of all transformations could at best be zero and with the slightest imperfection in engine function the sum would be less than zero – so heat would inevitably be lost and hence the process would not be reversible at all.

Clausius also came up with a word to describe *how* the sum of all transformations would always be less than zero. He called it *en tropie* – Greek for intrinsic direction, which is now captured in the second law of thermodynamics, which states that all things tend towards thermal equilibrium by the transfer of heat from hotter things to cooler things, not the other way around.

Clausius further modified his inequality equation to provide a way of calculating by just how much heat loss occurred in each engine cycle, which he called delta s – where s is entropy and delta s, the change in entropy, equals the integral of all changes in temperature within the engine, delta Q, divided by the final temperature of the heat sink, T.

$$\Delta S = \oint \frac{\delta Q}{T}$$

This essentially generalizes the Carnot heat engine formula so that the concept can be applied to any process in the Universe. So heat will always move from hot things to cold things and when you try to harness that heat to do work, you'll inevitably lose a bit of it. The release of such unharnessed heat will eventually lead to a state of universal thermal equilibrium, where no temperature differences remain to drive any kind of work – a likely future for our Universe that cosmologists call its heat death.